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A Discrete Modeling Approach for Buck Converter

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Abstract

In this paper, a discrete modeling approach for Buck converters based on continuous condition mode (CCM) and discontinuous condition mode (DCM) was presented. The unified coefficient matrixes of discrete model were described by building a mathematical function and the calculation methods of the parameters in coefficient matrixes were given. The working states of Buck converter on various work conditions were described adopting one discrete equation. The validity of the proposed modeling approach was proved by contrasting the output of discrete model with the operation result of Buck converter system in Simulink.

Keywords:discrete model; Buck; continuous condition mode; discontinuous condition mode; united coefficient

1.Introduction

Buck converter is an essential DC-DC converter. For high efficiency, Buck converter is widely applied to electron system, for example computer with battery supply, mobile telephone.

Buck converter is inherency non-linear system, the research and control of its non-linear phenomena is an important branch of power electronics. The mathematical model of converter is an effective tool which is adopted to research the operation characteristics and control methods of converter[1-2]. The modeling method of DC-DC converter undergoes a developmental process from linearity modeling to non-linearity modeling. In literature [3] and [4], it is mentioned that the most appropriate model for the research of non-linear phenomena in DC-DC converter is discrete model according to correlative research results. In recent years, some scholars have deduced the discrete model of DC-DC converter in different working state based on discrete time mapping method, and the research of bifurcation and chaos phenomena in converter has been done based on the founded discrete model[5-7].

According to the operation state of inductor current, the operation mode of converter divides into two modes, which are continuous conduction mode (CCM) and discontinuous conduction mode (DCM). The known discrete mapping models of DC-DC converter are founded based on a certain operation mode[7-10], that is, before founding model, it is supposed that the operation mode of converter is one certain mode. But,

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the operation mode of converter doesn't maintain one mode during the large signal operation process of DC-DC converter. So, it is significant that a discrete model of converter is established based on both CCM and DCM. The discrete model can accurately describe the states of converter on various operation conditions; it can provide the mathematics model for designing converter digital controller, forecasting system operation states and analyzing operation characteristics etc.

In this paper, a discrete modeling approach for Buck converters based on CCM and DCM is presented.

2.Discrete modeling method

2.1.Discrete Model Expression of Buck Converter

Fig. 1 shows the circuit topology of Buck converter, S is power switch, VD is diode, L is inductor, C is capacitance, V_{in} is input voltage, i_L is inductor current, v_C is capacitance voltage, v_o is output voltage, R is load.

Fig.2 shows continuous inductor current and discontinuous inductor current respectively, $t_{on}=d\mathrm{T}$ is turning on time of power switch, d is duty ration, T is switching period, $t_{off}=(1-d)\mathrm{T}$ is turning off time, Δt is duration time of inductor current after power switch turns off.

Equation (1) describes the three linear operation states when Buck converter operates in DCM.

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{A}_1\boldsymbol{x} + \boldsymbol{B}_1 V_{in} \\ \dot{\boldsymbol{x}} = \boldsymbol{A}_2\boldsymbol{x} + \boldsymbol{B}_2 V_{in} \\ \dot{\boldsymbol{x}} = \boldsymbol{A}_3\boldsymbol{x} \end{cases} \tag{1}$$

here, state variable $x=[v_C, i_L]^{\mathrm{T}}$, the coefficient matrixes are as follows.

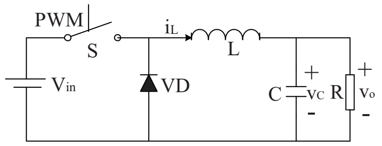

Figure 1. The circuit topology of Buck converter

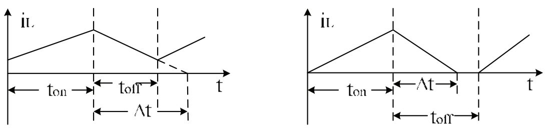

Figure 2. The sketch of inductance current

$$\boldsymbol{A}_1 = \boldsymbol{A}_2 = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}, \boldsymbol{B}_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \boldsymbol{B}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \boldsymbol{A}_3 = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix}.$$

Equation (1) can be transformed into (2) by adopting state transfer matrix.

$$\begin{cases} \boldsymbol{x}_{(k+d_k)T} = e^{\boldsymbol{A}_1 d_k T}\boldsymbol{x}_{kT} + \int_{kT}^{kT+d_kT} e^{\boldsymbol{A}_1(kT+d_kT-\tau)}\boldsymbol{B}_1 V_{in} d\tau \\ \qquad = \boldsymbol{G}_{D1}\boldsymbol{x}_{kT} + \boldsymbol{H}_{D1} \\ \boldsymbol{x}_{(k+d_k)T+\Delta t} = e^{\boldsymbol{A}_2\Delta t}\boldsymbol{x}_{(k+d_k)T} + \int_{kT+d_kT}^{kT+d_kT+\Delta t} e^{\boldsymbol{A}_2(kT+d_kT+\Delta t-\tau)}\boldsymbol{B}_2 V_{in} d\tau \\ \qquad = \boldsymbol{G}_{D2}\boldsymbol{x}_{(k+d_k)T} + \boldsymbol{H}_{D2} \\ \boldsymbol{x}_{(k+1)T} = e^{\boldsymbol{A}_3(T-d_kT-\Delta t)}\boldsymbol{x}_{(kT+d_kT+\Delta t)} \\ \qquad = \boldsymbol{G}_{D3}\boldsymbol{x}_{(k+d_k)T+\Delta t} \end{cases} \tag{2}$$

d_k is the duty ratio of the kth switching period. The discrete model expression of Buck converter operating in DCM can be deduced by incorporating the three equations of (2), the result is:

$$\boldsymbol{x}_{(k+1)T} = \boldsymbol{G}_{D3}\boldsymbol{G}_{D2}\boldsymbol{G}_{D1}\boldsymbol{x}_{kT} + \boldsymbol{G}_{D3}\boldsymbol{G}_{D2}\boldsymbol{H}_{D1} + \boldsymbol{G}_{D3}\boldsymbol{H}_{D2}. \tag{3}$$

When Buck converter operates in CCM, its two linear operation states are described by the first and second equations of (1), then the corresponding expressions of state matrix are:

$$\begin{cases} \boldsymbol{x}_{(k+d_k)T} = e^{\boldsymbol{A}_1 d_k T}\boldsymbol{x}_{kT} + \int_{kT}^{kT+d_kT} e^{\boldsymbol{A}_1(kT+d_kT-\tau)}\boldsymbol{B}_1 V_{in} d\tau \\ \qquad = \boldsymbol{G}_{C1}\boldsymbol{x}_{kT} + \boldsymbol{H}_{C1} \\ \boldsymbol{x}_{(k+1)T} = e^{\boldsymbol{A}_2(T-d_kT)}\boldsymbol{x}_{(k+d_k)T} + \int_{kT+d_kT}^{kT+T} e^{\boldsymbol{A}_2(kT+T-\tau)}\boldsymbol{B}_2 V_{in} d\tau \\ \qquad = \boldsymbol{G}_{C2}\boldsymbol{x}_{(k+d_k)T} + \boldsymbol{H}_{C2} \end{cases} \Rightarrow$$

$$\boldsymbol{x}_{(k+1)T} = \boldsymbol{G}_{C2}\boldsymbol{G}_{C1}\boldsymbol{x}_{kT} + \boldsymbol{G}_{C2}\boldsymbol{H}_{C1} + \boldsymbol{H}_{C2}. \tag{4}$$

For founding the discrete expression of Buck converter based on the two operation modes (CCM and DCM), (4) is rewritten as:

$$\begin{aligned} \boldsymbol{x}_{(k+1)T} &= \boldsymbol{I}\boldsymbol{G}_{C2}\boldsymbol{G}_{C1}\boldsymbol{x}_{kT} + \boldsymbol{I}\boldsymbol{G}_{C2}\boldsymbol{H}_{C1} + \boldsymbol{I}\boldsymbol{H}_{C2} \\ &= \boldsymbol{G}_{C3}\boldsymbol{G}_{C2}\boldsymbol{G}_{C1}\boldsymbol{x}_{kT} + \boldsymbol{G}_{C3}\boldsymbol{G}_{C2}\boldsymbol{H}_{C1} + \boldsymbol{G}_{C3}\boldsymbol{H}_{C2} \end{aligned}. \tag{5}$$

$\boldsymbol{I}$ is unit matrix.

Then, the discrete model of Buck converter operating in CCM or DCM has the same mathematics expression. Next, the coefficient matrixes of expression will be united.

Equation (6) shows a function structured based on symbol function.

$$\begin{aligned} & f[\mathrm{sgn}(\Delta t - t_{off})] = \frac{1}{2}\mathrm{sgn}(\Delta t - t_{off})[\mathrm{sgn}(\Delta t - t_{off}) - 1] \\ & = \begin{cases} 0, & \Delta t \ge t_{off} \\ 1, & \Delta t < t_{off} \end{cases} \end{aligned} \tag{6}$$

sgn( ) is symbol function.

$\boldsymbol{G}_{C2}$ and $\boldsymbol{G}_{D2}$, $\boldsymbol{G}_{C3}$ and $\boldsymbol{G}_{D3}$, $\boldsymbol{H}_{C1}$ and $\boldsymbol{H}_{D1}$, $\boldsymbol{H}_{C2}$ and $\boldsymbol{H}_{D2}$ can be respectively united adopting expression (6), G_{C1}andG_{D1} is the same. $\boldsymbol{G}_1$, $\boldsymbol{G}_2$, $\boldsymbol{G}_3$and $\boldsymbol{H}_1$, $\boldsymbol{H}_2$ are the coefficient matrixes united, which are as follows:

$$\boldsymbol{G}_1 = e^{\boldsymbol{A}_1 d_k T}, \boldsymbol{G}_2 = e^{\boldsymbol{A}_2\left\{\Delta t f[\mathrm{sgn}(\Delta t - t_{off})] + \{1 - f[\mathrm{sgn}(\Delta t - t_{off})]\}(T - d_k T)\right\}}, \boldsymbol{G}_3 = e^{\boldsymbol{A}_3 f[\mathrm{sgn}(\Delta t - t_{off})](T - d_k T - \Delta t)},$$

$$\boldsymbol{H}_1 = \int_{kT}^{kT+d_kT} e^{\boldsymbol{A}_1(kT+d_kT-\tau)} \boldsymbol{B}_1 V_{in} d\tau, \boldsymbol{H}_2 = \int_{kT+d_kT}^{kT+T+[\Delta t-(T-d_kT)]f[\mathrm{sgn}(\Delta t - t_{off})]} e^{\boldsymbol{A}_2\{kT+T+[\Delta t-(T-d_kT)]f[\mathrm{sgn}(\Delta t - t_{off})]-\tau\}} \boldsymbol{B}_2 V_{in} d\tau \cdot \quad (7)$$

So, the discrete model of Buck converter based on both CCM and DCM is shown in (8), whose coefficient matrixes are described in (7).

$$\boldsymbol{x}_{(k+1)T} = \boldsymbol{G}_3\boldsymbol{G}_2\boldsymbol{G}_1\boldsymbol{x}_{kT} + \boldsymbol{G}_3\boldsymbol{G}_2\boldsymbol{H}_1 + \boldsymbol{G}_3\boldsymbol{H}_2 \quad (8)$$

2.2.Calculation Method of the Parameters in Model Coefficient Matrixes

In modeling process, there are two main steps need to be fulfilled. One is the solution of power-exponent function, the other is the calculation of Δt in a switching cycle.

- The power-exponent function can be calculated adopting Cayley-Hamilton.

According to the eigenvalues of matrix $\boldsymbol{A}_1$, the calculation of $e^{\boldsymbol{A}_1 d_k T}$ has three situation:1)when $L<4R^2C$, the eigenvaluea of $\boldsymbol{A}_1$ are conjugated complex number; 2) when $L>4R^2C$, the eigenvaluea of $\boldsymbol{A}_1$ are two different real number; 3) when $L=4R^2C$, the eigenvaluea of $\boldsymbol{A}_1$ are two same real number. Because the calculation involves trigonometric function, the first situation is most complicated. In this paper, $e^{\boldsymbol{A}_1 d_k T}$ is solved on the condition $L<4R^2C$.

$$\boldsymbol{G}_1 = \alpha \begin{bmatrix} \cos(\omega t_1) + \dfrac{\sigma \sin(\omega t_1)}{\omega} & \dfrac{\sin(\omega t_1)}{\omega C} \\ -\dfrac{\sin(\omega t_1)}{\omega L} & \cos(\omega t_1) - \dfrac{\sigma \sin(\omega t_1)}{\omega} \end{bmatrix} \quad (9)$$

here, $\sigma = -\dfrac{1}{2RC}, \omega = \dfrac{1}{2}\sqrt{\dfrac{4}{LC} - \dfrac{1}{R^2C^2}}, \alpha = e^{\sigma t_1}, t_1 = d_k T$.

Because $\boldsymbol{A}_2$=$\boldsymbol{A}_1$, $\boldsymbol{G}_2$ and $\boldsymbol{G}_1$ have same expression form.

$$\boldsymbol{G}_2 = \beta \begin{bmatrix} \cos(\omega t_2) + \dfrac{\sigma \sin(\omega t_2)}{\omega} & \dfrac{\sin(\omega t_2)}{\omega C} \\ -\dfrac{\sin(\omega t_2)}{\omega L} & \cos(\omega t_2) - \dfrac{\sigma \sin(\omega t_2)}{\omega} \end{bmatrix} \quad (10)$$

here, $\beta = e^{\sigma t_2}, t_2 = \Delta t\, f[\mathrm{sgn}(\Delta t - t_{off})] + \{1 - f[\mathrm{sgn}(\Delta t - t_{off})]\}(T - d_k T)$.

$$\boldsymbol{G}_3 = \begin{bmatrix} e^{-\frac{1}{RC}t_3} & 0 \\ 0 & 1 \end{bmatrix} \quad (11)$$

$\boldsymbol{H}_1$and $\boldsymbol{H}_2$ is respectively solved according to (7).

$$\boldsymbol{H}_1 = (e^{\boldsymbol{A}_1 d_k T} - \boldsymbol{I})\boldsymbol{A}_1^{-1}\boldsymbol{B}_1 V_{in}\ ,\ \ \boldsymbol{H}_2 = 0 \tag{12}$$

- the solving method of Δt

Δt is the needed time during which inductor current decreases to zero after power switch turns off, so Δt can be solved by supposing inductor current naturally decreases to zero after power switch turns off, namely, supposing the current of $\boldsymbol{x}_{(k+d_k)\mathrm{T}+\Delta t}$ in (2) is zero. The current expression of $\boldsymbol{x}_{(k+d_k)\mathrm{T}+\Delta t}$ is:

$$\begin{aligned} i_{(k+d_k)T+\Delta t} &= e^{-\frac{\Delta t}{2RC}}[\delta_1 \sin(\omega\Delta t) + \delta_2 \cos(\omega\Delta t)] \\ &= \rho e^{-\frac{\Delta t}{2RC}} \sin(\omega\Delta t + \theta) \\ &= y(\Delta t) \end{aligned} \tag{13}$$

here $\rho = \sqrt{\delta_1^2 + \delta_2^2}$, $\theta = arctg(\delta_2/\delta_1)$, $\delta_2 = i_{L(k+d_k)T}$, $\delta_1 = \dfrac{i_{L(k+d_k)T}}{2RC\omega} - \dfrac{v_{C(k+d_k)T}}{\omega L}$. $v_{C(k+d_k)T}$ and $i_{L(k+d_k)T}$ is respectively the capacitance voltage and inductor current when power switch begins to turn off during the kth period. In feed-back closed loop system of Buck converter, the duty ratio d_k of the kth period can be ascertained according to control strategy and state sampling values before the kth period, then $v_{C(k+d_k)T}$ and $i_{L(k+d_k)T}$ can be educed according to the first equation of (2). So, (13) has only one unknown variable Δt.

The value of Δt can be calculated by setting (13) equaling to zero.

$$\Delta t = \frac{\pi - \theta}{\omega} \qquad (\frac{\pi}{2} \le \theta < \pi) \tag{14}$$

3.Simulation results

For validating the correctness of discrete model established in this paper, the operation results of simulation model in Simulink are compared with the operation results of discrete model, Buck converter feed-back closed loop system parameters are as follows: V_{in}=60V,L=800μH,C=100μF , R=2**Ω** ,T=10μs, V_m=5V, V_m is the amplitude of saw-tooth wave, V_{ref}=5V, V_{ref} is reference voltage. So the expression of duty ratio d is $d = 1-(gv_C - V_{ref})/V_m$, here 0<d<1, v_C is capacitance voltage and g is feed-back coefficient, the inductor value in this set converter parameters satisfies the operation condition for CCM: $L > R(1-d)/2f$.

Fig. 3 shows the operation results of Buck feed-back closed loop system in Simulink, (a) shows steady operation with continuous inductor current, (b) shows an operation state with big oscillatory phenomenon, capacitance voltage curve is signed with symbol 1 and inductor current curve is signed with symbol 2 in fig. 3. According to fig. 3, inductor current isn't always continuous or discontinuous when system exhibits big oscillatory phenomenon, namely Buck system shows both CCM and DCM.

Buck converter will also show both CCM and DCM when converter parameters satisfy the operation condition for DCM and converter operates with large signal. For satisfying the operation condition for DCM, the inductor value in above-mentioned parameter is adjusted, choosing L=2μH, fig. 3(c) shows the start-up process of converter in Simulink. According to fig. 3(c), the operation mode of Buck converter isn't only one when converter operates from start-up to steady, the operation process is from CCM to DCM.

Fig. 4 show the operation results of Buck converter system based on discrete model, the work conditions are the same as those conditions adopted in Simulink, (a) shows steady operation with continuous inductor current, (b) shows operation state with big oscillatory phenomenon, (c) shows the start-up process when the steady current of system is discontinuous.

The results in fig. 4 are accordant with the results in fig. 3, so the discrete model of Buck converter established in this paper can accurately describe system characteristics on different operation conditions.

4.Conclusions

A discrete modeling approach for Buck converters based on CCM and DCM is presented and the calculation methods of model parameters are given in this paper. The working states of Buck converter on various operation conditions are described adopting one discrete equation, which provides the mathematics model for designing converter digital controller, forecasting system operation states and analyzing operation characteristics etc.

The validity of the founded discrete model is proved by contrasting the output of model with the operation result of Buck converter system in Simulink.

The main work of this paper is to discuss the modeling method of Buck converter, so, circuit components of converter are ideal components. The parasitic parameters of components will not affect the validity of modeling method and the presented method can be applied to other DC-DC converter.

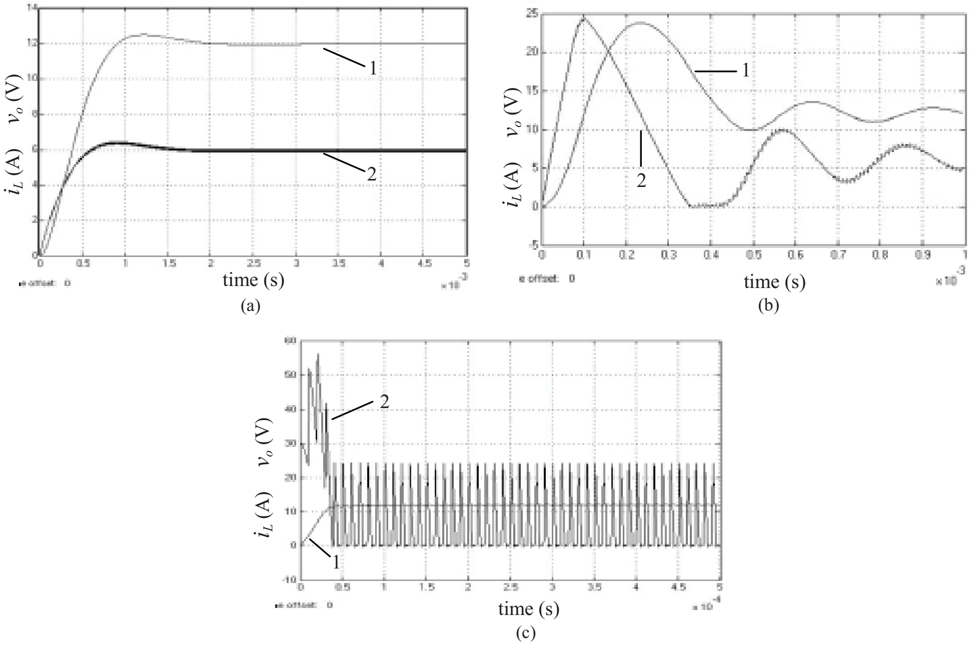

Figure 3. The simulation results of Buck feed-back closed-loop system in Simulink

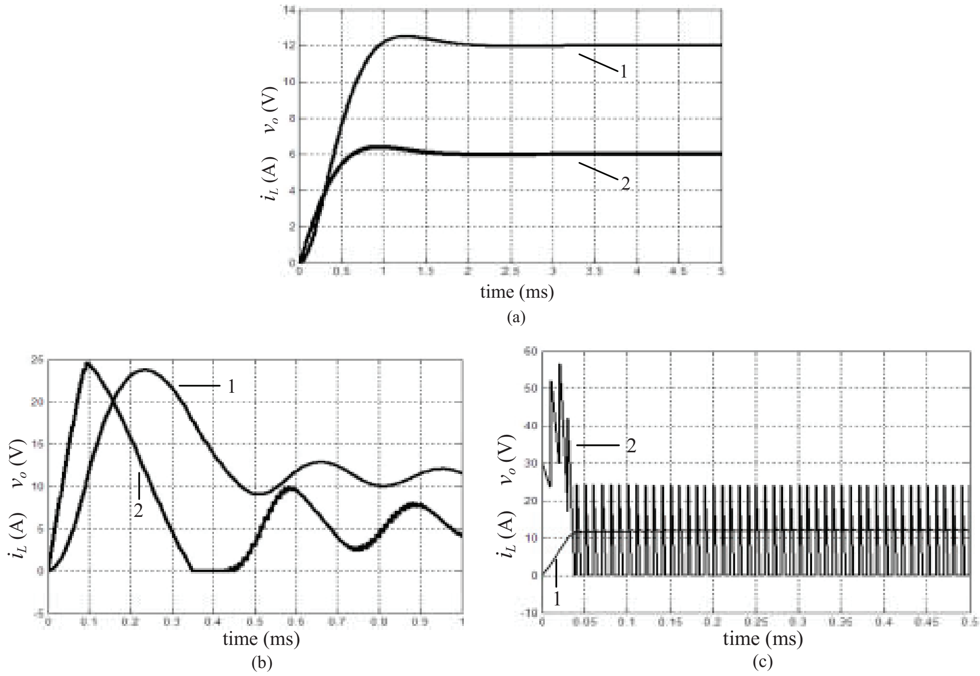

Figure 4. The simulation results of Buck feed-back closed-loop system based on discrete model

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